07/5202 Efficient Usage of Self Validated Integrators for Space Applications

Type of activity: Extended Study (6 months, 35 KEUR)

Background and Motivation

Several space related problems can be formulated as ordinary differential equations. Even in the simplest cases analytical solutions do not exist, thus numerical techniques are required. These use floating point calculations and thus introduce rounding errors which may lead to unacceptable inaccuracies and ultimately to mission failure. Applications such as space surveillance, Near Earth Objects hazard mitigation, collision avoidance manoeuvres, would benefit from approaches where predictions can mathematically be proven to be correct.

Self validated integration methods [5] based on interval mathematics seem to be promising techniques with this respect. The inclusion of the real solution of the problem considered is provided by these methods with mathematical rigor. However, the resulting inclusion could have overestimation, depending on the problem description and the method used to obtain the enclosure of the function. Reasonably tight inclusion of the solution needs some analysis (i.e. investigation of suitable formalism) and experiment on the problem and application of the proper techniques.

Many efficient enclosure methods and accelerating tools have been invented recently. There are general solvers available (VNODE [11], COSY [4], AWA [1], ValEncIA-IVP [10], VSPODE [6]) applying these methods to solve ODE problems in reliable way. The results of a past Ariadna study [2] show that using these off-the-shelf solvers (especially AWA and VNODE) applied for space flight mechanical problems could lead pessimistic results in terms of large overestimation in the enclosure intervals of the solution.

Recent results show that validated ODE solvers based on Taylor models [3, 7], or based on calculation of state enclosures consisting of non-validated approximate solutions of the problem together with guaranteed error bounds for the approximation errors [10] are able to perform very well even on difficult problems ([10] and the references there, also [8,9]). These studies show that self validated solvers are able to reduce the wrapping effect and the dependency problem. The aim of this study is to investigate these new techniques and develop an efficient solver to have reasonable tight inclusion of the solution of selected space flight mechanics problems. To reach that point, a deep analysis on why the general interval solvers fail so early is needed.

Research and Study Objective

- Review the state-of-the-art of rigorous numerical techniques.
- Choice of a space related problem for which these techniques are sought to be promising. (Suggestion: the dynamic of a point mass in a gravitational field using intervals as initial conditions and as perturbations, and using Cartesian coordinates.)
  Development of a validated ODE solver based on Taylor models, with accelerating tools exploiting the selected problem structure.
- Detailed testing of the developed solver on selected problem instances.

It is emphasized that for this study space knowledge is not required (will be provided by the ACT) and the non-space academic community is strongly encouraged to tender.

The study will be performed in close cooperation with the ACT researchers.

References

[1] AWA (http://www.math.uni-wuppertal.de/~xsc/xsc/pxsc_software.html#awa)

[2] F. Bernelli Zazzera, M. Vasile, M. Massari, P. Di Lizia. Assessing the accuracy of interval arithmetic estimates in space flight mechanics. Final report, Ariadna id: 04/4105, Contract Number: 18851/05/NL/MV, 2004. (http://www.esa.int/gsp/ACT/doc/ACT-RPT-04-4105-ARIADNA-Assessing_the_Accuracy_of_Interval_arithmetic_Estimates.pdf)

[3] M. Berz. From Taylor series to Taylor models. In AIP Conference Proceedings 405, pages 1–23, 1997.

[4] COSY (http://bt.pa.msu.edu/index_cosy.htm)

[5] G. F. Corliss. Guaranteed error bounds for ordinary differential equations. In Ainsworth, Levesley, Light and Marletta, editors, Theory of Numerics in Ordinary and Partial Differential Equations, pages 1-75. Oxford University Press, 1995.

[6] Y. Lin and M. A. Stadtherr, "Validated Solutions of Initial Value Problems for Parametric ODEs," to appear in Applied Numerical Mathematics, 2007.

[7] K. Makino and M. Berz. Taylor models and other validated functional inclusion methods. Int. J. Pure Appl. Math., 4:379–456, 2003.

[8] M. Neher. Verified integration of linear n-th order ODEs using large steps. Preprint 06/20, Fakultät für Mathematik, Universität Karlsruhe, 2006.

[9] M. Neher, K. R. Jackson and N. S. Nedialkov. On Taylor model based integration of ODEs. Preprint 05/21, Fakultät für Mathematik, Universität Karlsruhe, 2005.

[10] ValEncIA-IVP (http://www.valencia-ivp.com/)

[11] VNODE (http://www.cas.mcmaster.ca/~nedialk/Software/VNODE/VNODE.shtml)